DISCRETE MATHEMATICS

QUESTION BANK

ADVANCED LEARNER

1) Define following:
a) Power set
b) Relation
c) Function
d) Symmetric difference of sets
e) Cartesian product of sets

2) Determine whether the following argument form is valid or invalid by drawing a truth table:

$$p \wedge q \rightarrow \backsim r$$
$$p \vee \backsim q$$
$$\backsim q \rightarrow p$$
$$\therefore \backsim r$$

3) Let P(x) be the predicate "x > 1/x."
a) Write P(2), P(1/2), P(−1), P(−1/2), and P(−8), and indicate which of these statements are true and which are false.
b) Find the truth set of P(x) if the domain of x is **R**, the set of all real numbers
c) If the domain is the set **R**+ of all positive real numbers, what is the truth set of P(x)?

4) Prove statements directly from the definition of divisibility
a) For all integers a, b, and c, if $a \mid b$ and $a \mid c$ then $a \mid (b - c)$.
b) For all integers a, b, and c, if $a \mid b$ and $a \mid c$ then $a \mid (b + c)$.

5) Using backtracking method, solve $a_n = 7a_{n-1}$ for $n \geq 1$ and $a_2 = 98$.
6) Use characteristic root method to solve the given recurrence relation
$$a_n = 4a_{n-1} - 4a_{n-2} = 0. \, with \, intial \, condition \, a_0 = 1 \, \& \, a_1 = 1$$

7)
a) Define a relation R on **R** (the set of all real numbers) as follows: For all real numbers x and y. x R y ⟺ x = y.
b) Define a relation R on **R** (the set of all real numbers) as follows: For all x, y ∈ R, x R y ⟺ x < y.
Is R reflexive? b. Is R symmetric? c. Is R transitive?

8) In the graph below, determine which of the following walks are trails, paths, circuits or simple circuits.
i. $v_1 e_1 v_2 e_3 v_3 e_4 v_3 e_5 v_4$
ii. $e_1 e_3 e_5 e_5 e_6$
iii. $v_2 v_3 v_4 v_5 v_3 v_6 v_2$
iv. $v_1 e_1 v_2 e_1 v_1$
v. v_1
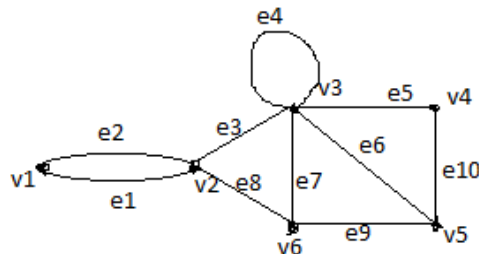
9) Suppose there is a co – education school having 60 % boys and 40 % girls as students. The girl students have secured first class and second class in equal numbers and all boys secured first class. A teacher selects a student at random and verifies the result. Teacher observes that a student has secured first class in examination. What is the probability that the student is a girl?

10) An urn contains 5 blue and 7 gray balls. Let us say that 2 are chosen at random, one after the other, without replacement.

a) Find the following probabilities that both balls are blue, the probability that the first ball is blue and the second is not blue, the probability that the first ball is not blue and the second ball is blue, and the probability that neither ball is blue.

b) What is the probability that the second ball is blue?

SLOW LEARNER

1) Given sets A and B, the symmetric difference of A and B, denoted A Δ B, is
$$A \Delta B = (A - B) \cup (B - A).$$
Let A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, and C = {5, 6, 7, 8}. Find each of the following sets:
a) A Δ B
b) B Δ C
c) A Δ C
d) (A Δ B) Δ C

2) Using truth table, verify the following logical equivalence.
a) $p \leftrightarrow q \equiv \sim(p \wedge \sim q) \wedge \sim (q \wedge \sim p)$
b) $\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$
c) $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

3) Let D = E = {−2, −1, 0, 1, 2}. Write negations for each of the following statements:
a) ∀ x in D, ∃y in E such that x + y = 1.
b) ∃x in D such that ∀ y in E, x + y = −y.
c) ∀x in D, ∃y in E such that x.y ≥ y.
d) ∃x in D such that ∀y in E, x ≤ y.
4) Prove that for all integers m and n, m + n and m − n are either both odd or both even.[
5) Find the following summation:
a) $\sum_{k=1}^{5} 2^k$

b) $\sum_{k=0}^{5} k^2$

c) $\sum_{k=1}^{3} (-1)^k / k$

d) $\sum_{k=1}^{6} (k + 1)^3$

6) Define composite function and find fog(3) and gof(3) where, $f(x) = 3x^2 - 1$ and $g(x) = 5-2x$

7) Let A = {0, 1, 2, 3, 4} and define a relation R on A as follows:
R = {(0, 0), (0, 4), (1, 1), (1, 3), (2, 2), (3, 1), (3, 3), (4, 0), (4, 4)}.
R is an equivalence relation on A. Find the distinct equivalence classes of R.

8) Define the following:
a)   trail
b)   Walk
c)   Path
d)   Circuits
e)   Cycle

9) A coin is loaded so that the probability of heads is 0.6. Suppose the coin is tossed twice.
a) What is the probability of obtaining two heads?
b) What is the probability of obtaining one head?
c) What is the probability of obtaining no heads?
d) What is the probability of obtaining at least one head?

10) A drawer contains ten black and ten white socks. You reach in and pull some out without looking at them. What is the *least* number of socks you must pull out to be sure to get a matched pair? Explain how the answer follows from the pigeonhole principle.

ASSIGNMENT QUESTIONS

1) Determine the number of integers from 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7.
2) Disprove the following by giving two counter examples:
    i.   For all real numbers a and b, if a < b then $a^2 < b^2$.
    ii. For all integers n, if n is odd then (n – 1)/2 is odd.
    iii. For all integers m and n, if 2m + n is odd then m and n are both odd.
    iv. ∀ real numbers a and b, if $a^2 = b^2$ then a = b.

3) Using principal of Mathematical Induction, prove that: $5^n – 1$ is divisible by 4.
4) Define a relation T on **Z** (the set of all integers) as follows: For all integers m and n,
$$m \text{ T } n \Leftrightarrow 3 \mid (m - n).$$
This relation is called **congruence modulo 3.** Prove that T is an equivalence relation.
5) Consider a medical test that screens for a disease found in 5 people in 1,000. Suppose that the false positive rate is 3% and the false negative rate is 1%. Then 99% of the time a person who has the condition tests positive for it, and 97% of the time a person who does not have the condition tests negative for it.
a) What is the probability that a randomly chosen person who tests positive for the disease actually has the disease?
b) What is the probability that a randomly chosen person who tests negative for the disease does not indeed have the disease?

****